Signals and Systems Summary

John C. Mu

December 22, 2011

1 Introduction

This document includes most of the important formulas and concepts to remember for the main signals and systems examiners. This document was originally grouped by examiner. However, this grouping has been removed. Hopefully this document will make your quals experience less stressful and less confusing.

Thanks to my quals group members for their many contributions to this document: Jaime, Shangping, Frank, Derek and Yu.

Please let me know if there are any errors/typos in this document. Suggestions are also welcome.

2 Probabilty

2.1 Derived distributions

$$Y = g(X) \tag{1}$$
$$F_Y(y) = Pr(Y \le y) = Pr(g(X) \le y) = Pr(X \le g^{-1}(y)) \tag{2}$$

via CDF

$$f_Y(y) = \frac{d}{dy} F_Y(y) \tag{3}$$

General transformation formula

$$f_Y(y) = \sum_i \frac{f_X(x_i)}{|g'(x_i)|} \tag{4}$$

where x_i are the solutions to $y = g(x)$.

2.2 Problem Decomposition

- Independent dimensions
- Random process as difference equation

- Random process as sum of random variables, especially as sum of indicators

$$Y = \sum_i X_i, X_i \sim Bern(p_i) \tag{5}$$

$$E[Y] = E[\sum_i X_i] = \sum_i E[X_i] = \sum_i p_i \tag{6}$$

2.3 Misc. Formulas and Inequalities

Note that the variable names don't carry from line to line.

$$p(x) = \sum_y p(x|y)p_Y(y) \text{ or } = \int p(x|y)f_Y(y)dy \tag{7}$$

$$E[X] = E[E[X|Y]] \tag{8}$$

$$\text{Markov Ineq: } P(|X| > a) \leq \frac{E[X]}{a}, a > 0 \tag{9}$$

$$\text{Chebyshev Ineq:} P(|X - E[X]| \geq a) \leq \frac{Var(X)}{a^2} \tag{10}$$

$$\text{LLN: } \frac{1}{N}\sum_i X_i = E[X] \text{ , } X_i \text{iid} \tag{11}$$

$$\text{CLT: } \frac{\sum_i X_i - E[\sum_i X_i]}{\sqrt{N \cdot Var(X)}} \sim N(0,1) \text{ , } X_i \text{iid} \tag{12}$$

$$\text{Characteristic Function: } \phi(t) = E[e^{jtX}] \tag{13}$$

Poisson Process

3 Fourier

Common reductions

$$\frac{1}{N}\sum_{k=0}^{N-1} \exp(2\pi j \frac{k}{N} n) = \delta[n], \text{ similar expression for integral} \tag{14}$$

Z-Transform

4 General

Common Taylor expansions.

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{15}$$

$$log(1 - x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \tag{16}$$

$$f(x) = f(a) + \frac{f'(a)}{1!} + \frac{f''(a)}{2!} + \cdots \tag{17}$$

Cauchy-Schwartz
Jensen's Inequality
Circulant Matrix